ДИС Кирчев 5.11

Производна

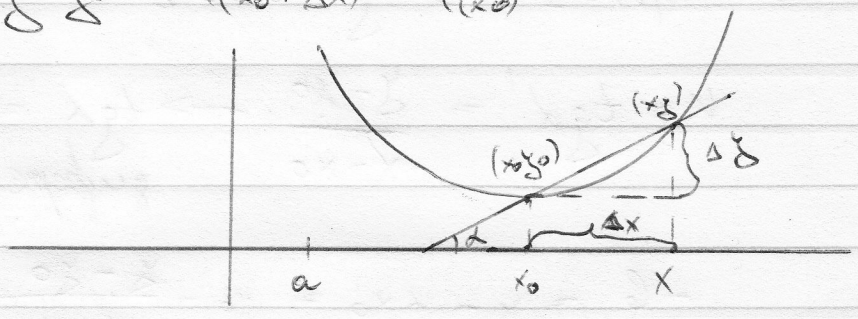
$f(x)$ $\quad x = x_0 + \Delta x$

$y_0 = f(x_0)$

$y = f(x) = f(x_0 + \Delta x)$

$\Delta y = y - y_0 = f(x_0 + \Delta x) - f(x_0)$

черт. 1

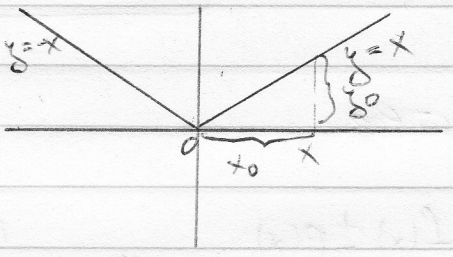


$$\Phi\left(tg\alpha = \frac{\Delta y}{\Delta x} = \frac{f(x) - f(x_0)}{x - x_0} = \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x} \right)$$

Дефиринциално частно

? Съществува ли $\lim\limits_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x) - f(x_0)}{\Delta x}$

$y = |x| \rightarrow$



Ако има производна то отговора е $f(x_0)$

$\frac{x - x_0}{\Delta x} = \frac{x - x_0}{x - x_0} = 1 \longrightarrow 1$ граница

$\frac{-x + x_0}{x - x_0} = -1 \longrightarrow -1$ граница

① В 0 не съществува

Ако съществува производна в (ab) то е диференцируема

[a b] a - лява граница, b - дясна граница

от черт. 1 $\Rightarrow y = kx + b$

$$k = tg\alpha' = \frac{y - y_0}{x - x_0} \longrightarrow tg\alpha = f(x_0)$$

диференциално частно

$$-b = y - kx_0 = y_0 - \frac{y - y_0}{x - x_0} \cdot x_0 \quad \left(\text{заместване } k = \frac{y - y_0}{x - x_0}\right)$$

$$y = \frac{y - y_0}{x - x_0} \cdot x + y_0 - \frac{y - y_0}{x - x_0} x_0 \Rightarrow b = y_0 - f'(x_0)x_0$$

$$y = f'(x_0)x + y_0 - f'(x_0)x_0$$

$$\boxed{y - y_0 = f'(x_0)(x - x_0)} \quad \Phi$$

Свойства:

$M_x = f(x) \pm g(x)$ $\quad r'$ - производна

$r'(x_0) = f(x_0) + g(x_0)$

$$\Phi \mid \frac{r(x) - r(x_0)}{\Delta x} = \frac{f(x) - f(x_0)}{\Delta x} \pm \frac{g(x) - g(x_0)}{\Delta x}$$

$$\Phi \mid f(x)g(x) = f'(x)g(x) + f(x)g'(x)$$

$$\frac{f(x)g(x) - f(x_0)g(x_0)}{\Delta x} \qquad \Delta x = x - x_0$$

$(\pm f(x_0)g(x)$ - добавяме $)$

$$\frac{f(x)g(x) - f(x_0)g(x) + f(x_0)g(x) - f(x_0)g(x_0)}{\Delta x} =$$

$$\frac{f(x) - f(x_0)}{\Delta x} g(x) + f(x_0)\frac{g(x) - g(x_0)}{\Delta x}$$

$\frac{f(x) - f(x_0)}{\Delta x} \to f'(x_0)$, $g(x) \to g(x_0)$, $f(x_0)$ остава, $\frac{g(x) - g(x_0)}{\Delta x} \to g'(x_0)$

$$\frac{f(x) - f(x_0)}{x - x_0} - f'(x_0) = \alpha \to 0$$

Φ $f(x) - f(x_0) = f'(x_0)(x - x_0) + \alpha(x - x_0)$ — число; $x - x_0 \to 0$, $\alpha \to 0$, $x - x_0 \to 0$

$\lim_{x \to x_0} f(x) = f(x_0)$ Непрекъснатост

$$\frac{f(x)}{g(x)} = \frac{1}{g^2(x)} \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}$$

$x = g(t)$ — D - дифференцируемое множество

$t \in D_1 \xrightarrow{g(t)} x \in D_2 \xrightarrow{f(x)} D_3$

$F(t) = f(g(t))$ $\qquad D_1 \xrightarrow{F(t)} D_2$

$\dfrac{g(t) - g(t_0)}{t - t_0} \longrightarrow g'(t_0)$

$\Delta t = t - t_0 \longrightarrow 0$

$x_0 = g(t_0)$

Φ $\boxed{F'(t_0) = f'(g(t_0))\, g'(t_0)}$ само при $\Delta x \neq 0$

$F(t) - F(t_0) = f(g(t)) - f(g(t_0)) = f(x) - f(x_0)$

но $f(x) = f(x_0) + f'(x_0)(x - x_0) + \alpha(x - x_0) \Rightarrow$

$f(x) - f(x_0) = \cancel{f(x_0)} + f'(x_0)(x - x_0) + \alpha(x - x_0) - \cancel{f(x_0)} =$

$f'(x_0)(x - x_0) + \alpha(x - x_0)$ / делим на $t - t_0$

$f'(x_0) \dfrac{g(t) - g(t_0)}{x - x_0} - \alpha \dfrac{g(t) - g(t_0)}{t - t_0}$

$\dfrac{g(t) - g(t_0)}{x - x_0} \Downarrow g'(t_0)$

$$\frac{\sin x - \sin x_0}{x - x_0} = \frac{1}{x - x_0} \; 2 \sin \frac{x - x_0}{2} \cos \frac{x + x_0}{2}$$

пояснение $t = \frac{x - x_0}{2}$

$$\left(\frac{\sin t}{t}\right) \cos\left(\frac{x - x_0}{2}\right) \longrightarrow \cos x_0$$

$\frac{\sin t}{t} \to 1$, $\frac{x - x_0}{2} \to x_0$

1) $(\sin x)' = \cos x$
2) $(\cos x)' = \sin\left(\frac{\pi}{2} + x\right) = -\sin x$
3) $\left(\frac{\pi}{2} + x\right)' = 1$
4) $\operatorname{tg} x = 1/\cos^2 x$
5) $\operatorname{ctg} x = -1/\sin^2 x$

$$\frac{x^n - x_0^n}{x - x_0} = \frac{(x - x_0)}{x - x_0} \underbrace{\left(x^{n-1} + x^{n-2} x_0 + x^{n-3} x_0^2 \; \cdots \; x_0^{n-1}\right)}_{n x_0^{n-1}}$$

$$\boxed{(x^n)' = n x_0^{n-1}} \quad \Phi$$